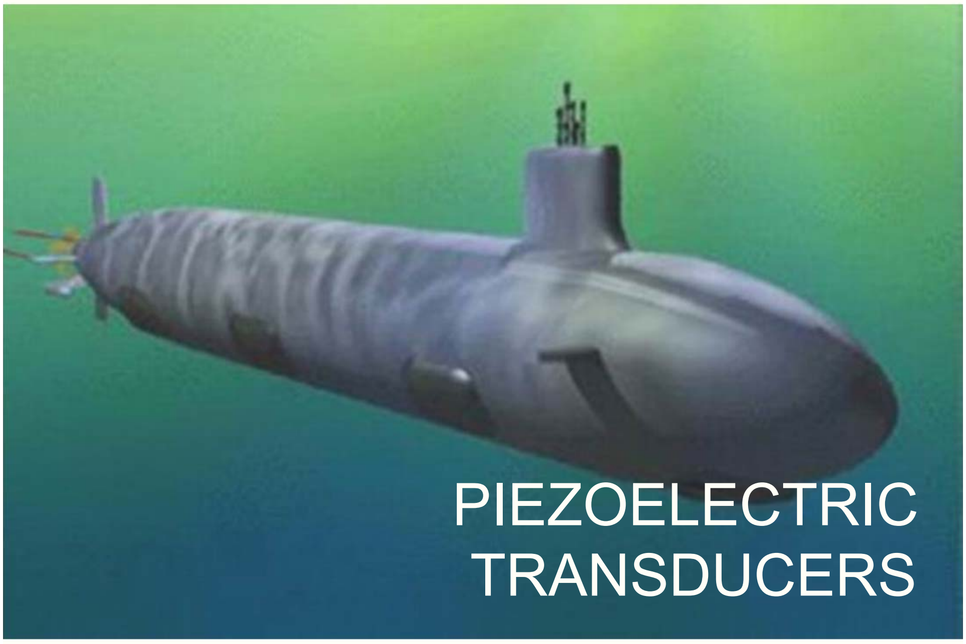
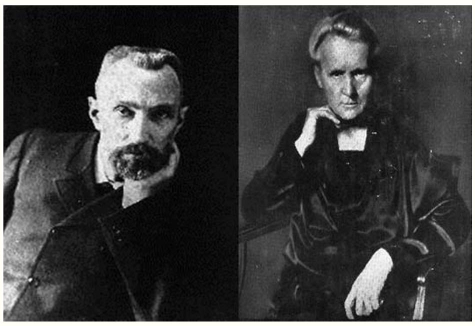




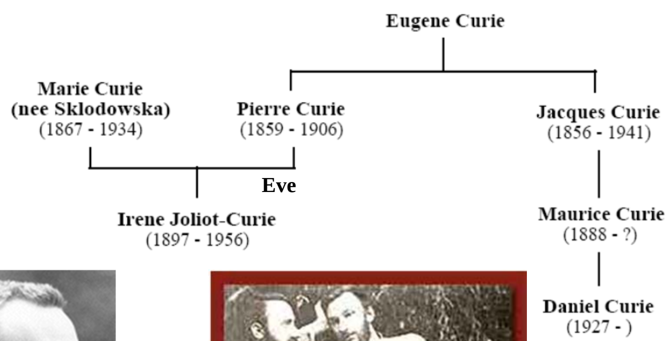
Pierre and Jacques Curie (1880)

- Introduction
- Electrostatics
- Piezoelectricity
- Signal conditioning
- Applications

Introduction

- 1880 Pierre and Jacques Curie: discovered the *piezoelectric effect* - mechanical stress induces surface electric charges
- 1881 Lippman: predicted the *converse piezoelectric effect* - electric field induces mechanical deformation
- Lord Kelvin, Pockels, Duhem, Voigt, R. E. Gibbs, Max Born: established theories and models
- Langevin: device for detecting submarines in World War I
- 1917 A. M. Nicolson: loud speakers, microphones, phonograph pickups, crystal oscillator
- 1921 Cady: Quartz crystal oscillators (GT cut)
- 1942 W. P. Mason: frequency filters
- 1950s~1960s R. D. Mindlin, H. F. Tiersten: vibration of piezoelectric plates

CURIE TREE



• Comparison of sensing principles

Principle	Strain Sensitivity (V/μ*)	Threshold (μ*)	Span to threshold ratio
Piezoelectric	5.0	0.00001	100.000.000
Piezoresistive	0.0001	0.0001	2.500.000
Inductive	0.001	0.0005	2.000.000
Capacitive	0.005	0.0001	750.000

Applications

High-voltage sources

Spark source
Transformer

Sensors

Microphone
Contact microphone
Microbalance
Accelerometer
Hydrophone

Actuators

Loudspeaker
Ultrasonic
Acousto-optic modulator
Inkjet head
Fuel injector

Frequency standard

Quartz resonator

Electrostatics

- Electric field

$$\mathbf{F} = q\mathbf{E}$$

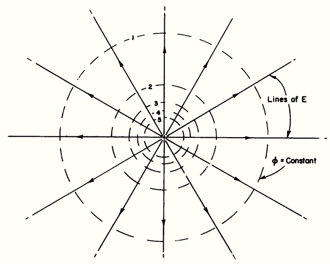
F: force
q: charge
E: electric field

- Coulomb's law

$$\mathbf{F} = kq_1q_2\frac{\mathbf{x}_1-\mathbf{x}_2}{|\mathbf{x}_1-\mathbf{x}_2|^3}$$

- Electric field of a point charge

$$\mathbf{E} = kq_1\frac{\mathbf{x}-\mathbf{x}_1}{|\mathbf{x}-\mathbf{x}_1|^3}$$



- Electric field due to a charge density $\rho(\mathbf{x})$

$$\mathbf{E} = k\int\rho(\mathbf{x}')\frac{\mathbf{x}-\mathbf{x}'}{|\mathbf{x}-\mathbf{x}'|^3}d\mathbf{x}'$$

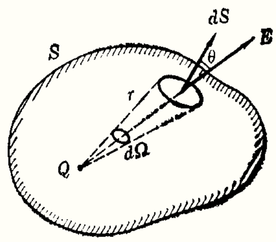
$$k \equiv \frac{1}{4\pi\varepsilon_0} = 10^{-7}c^2 = 8.988\times10^9\left(\frac{N\cdot m^2}{Coul^2}\right)$$

$$\varepsilon_0 = 8.8542\times10^{-12}\left(\frac{Coul^2}{N\cdot m^2}\right)$$: dielectric constant in vacuum

- Gauss's law

$$\oint\mathbf{E}\cdot\mathbf{n}ds = \frac{1}{\varepsilon_0}\int_V\rho(\mathbf{x})d\mathbf{x}$$

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0}$$



- Scalar potential

$$\mathbf{E} = -\nabla\varphi \quad , \quad \varphi(\mathbf{x}) = \frac{1}{4\pi\varepsilon_0}\int\frac{\rho(\mathbf{x}')}{|\mathbf{x}-\mathbf{x}'|}d\mathbf{x}' \quad , \quad \nabla\times\mathbf{E} = 0$$

φ : electrostatic potential

- Poisson and Laplace equations

$$\nabla\cdot\mathbf{E}=\frac{\rho}{\varepsilon_0}$$

$$\nabla\cdot(-\nabla\varphi)=\frac{\rho}{\varepsilon_0}$$

$$\nabla^2\varphi=-\frac{\rho}{\varepsilon_0}\ :\ \text{Poisson equation}$$

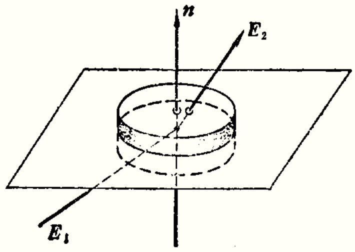
In regions of space where there is no charge density

$$\nabla^2\varphi=0\ :\ \text{Laplace equation}$$

- Boundary conditions

Dirichlet B.C. φ specified

Neumann B.C. $\frac{\partial\varphi}{\partial n}$ specified



- Discontinuities in the field and potential

$$(\mathbf{E}_2-\mathbf{E}_1)\cdot\mathbf{n}=\frac{\sigma}{\varepsilon_0}\quad ;\quad \varphi_1=\varphi_2$$

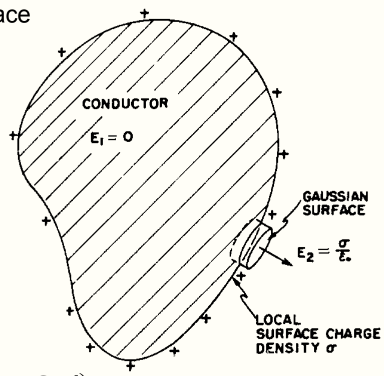
σ : surface-charge density ($Coul/m^2$)

- Conductors

Equipotential region, equipotential surface

$$\mathbf{E_1}=0\quad ;\quad \mathbf{E_2}=-\nabla\varphi$$

$$\nabla\varphi\cdot\mathbf{n}=\frac{\partial\varphi}{\partial n}\quad ;\quad \frac{\partial\varphi}{\partial n}=-\frac{\sigma}{\varepsilon_0}$$

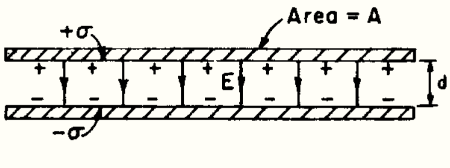


- Parallel-plate condenser

$$\varphi_1-\varphi_2=V\quad ,\quad V:\ \text{"voltage"}$$

$$V=Ed=\frac{\sigma}{\varepsilon_0}d=\frac{Q}{A}\frac{d}{\varepsilon_0}$$

$$C=\frac{Q}{V}=\frac{A\varepsilon_0}{d}\quad ,\quad C:\ \text{capatance}\left(Farad=\frac{Coul}{Volt}\right)$$

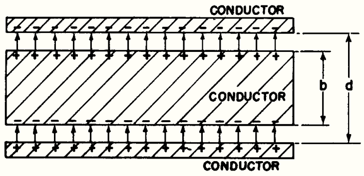


- Dielectrics

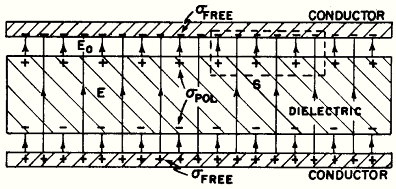
$$V = \frac{\sigma}{\varepsilon_0}(d-b)$$

$$C = \frac{\varepsilon_0 A}{d(1-b/d)}$$

The capacitance is increased.



Faraday discovered that the capacitance is increased when an insulator is put between the plates. $\Rightarrow$ The net charge inside the surface must be lower than it woiuld be without the dielectric.

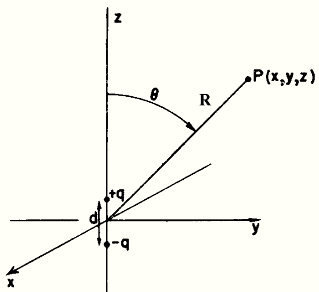


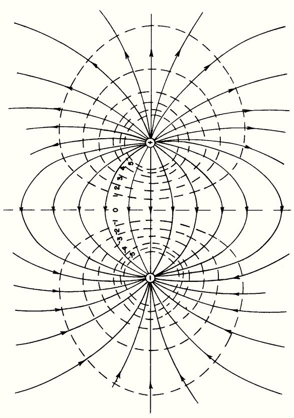
- Electric dipole

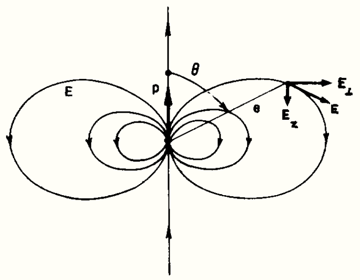
$p = qd$: dipole moment

$\mathbf{p}=q\mathbf{d}$, $\mathbf{d}$: points from $-q$ to $+q$

$\varphi(\mathbf{R}) = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\mathbf{R}}{R^3}$: dipole potential

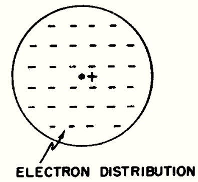




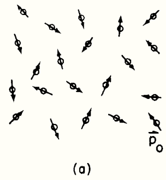


- The polarization vector **P**

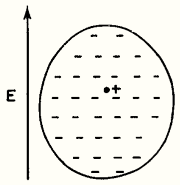
$Q = 0$
$\mathbf{p} = 0$



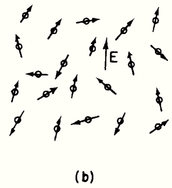
$Q = 0$
$\mathbf{p}_i \neq 0$
$\sum \mathbf{p}_i = 0$



$Q = 0$
$\mathbf{p} \neq 0$



$Q = 0$
$\mathbf{p}_i \neq 0$
$\sum \mathbf{p}_i \neq 0$



Polarization $\mathbf{P} = \dfrac{\sum \mathbf{p}_i}{\Delta v}$: dipole moment per unit volume

If the field is not too enormous $\mathbf{P} \propto \mathbf{E}$ (or $P_i = a_{ij} E_j$ for anisotropic materials)

- Polarization charges

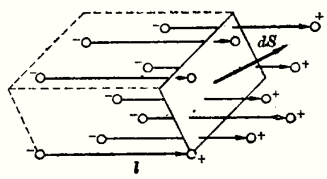
$\mathbf{p} = q\,\mathbf{l}$: dipole moment per molecule

$\mathbf{l}$: effective movement of positive charges w.r.t. negative charges

q: number of positive charges per molecule

$N = \dfrac{\text{number of molecules}}{\text{unit volume}}$

$Nq\mathbf{l} \cdot d\mathbf{S} = N\mathbf{p} \cdot d\mathbf{S} = \mathbf{P} \cdot d\mathbf{S}$: positive charges leave through $d\mathbf{S}$



1. Surface polarization charge density

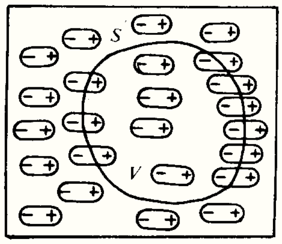
$\sigma_p = \dfrac{\mathbf{P} \cdot d\mathbf{S}}{dS} = \mathbf{n} \cdot \mathbf{P}$: *surface polarization charge density*

On the interface of two materials

$$\sigma_p = \mathbf{n} \cdot (\mathbf{P}_1 - \mathbf{P}_2)$$

2. Polarization charge density

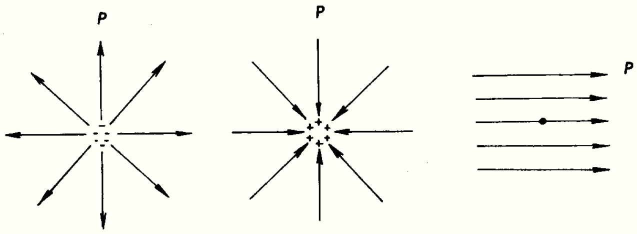
$\oint \mathbf{P}\cdot d\mathbf{S}$: total positive charges leave volume V
= net negative charges in volume V
$= -\int_V \rho_p dV$



ρ_p : polarization charge density

$$\oint \mathbf{P}\cdot d\mathbf{S} + \int_V \rho_p dV = 0$$

$$\int_V (\nabla\cdot\mathbf{P} + \rho_p) dV = 0 \qquad \rho_p = -\nabla\cdot\mathbf{P}$$



- Inside dielectric

$$\nabla\cdot\mathbf{E} = \frac{\rho_f + \rho_p}{\varepsilon_0}, \qquad \rho_f \text{ : free charge density}$$

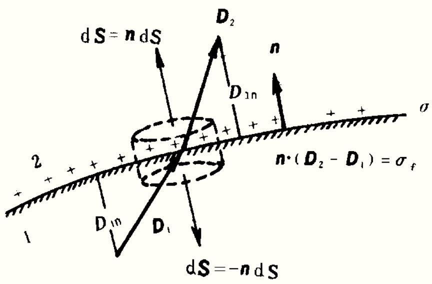
$$\varepsilon_0 \nabla\cdot\mathbf{E} = \rho_f + \rho_p = \rho_f - \nabla\cdot\mathbf{P}$$

or

$$\nabla\cdot(\varepsilon_0\mathbf{E} + \mathbf{P}) = \rho_f$$

Define "electric displacement"

$$\mathbf{D} = \varepsilon_0\mathbf{E} + \mathbf{P}$$



Governing equations

$\nabla\cdot\mathbf{D} = \rho_f$ and

$\nabla\times\mathbf{E} = 0$

- Constitutive equation

Isotropic materials

$$\mathbf{P} = \chi_e \varepsilon_0 \mathbf{E}$$

$$\mathbf{D} = (1+\chi_e)\varepsilon_0 \mathbf{E} = \varepsilon_r \varepsilon_0 \mathbf{E} = \varepsilon \mathbf{E}$$

χ_e : susceptibility
ε_0 : permittivity of empty space
ε_r : dielectric constant (relative permittivity)
ε : permittivity

Anisotropic materials

$$P_i = \varepsilon_0 \chi_{ij} E_j$$

$$D_i = \varepsilon_0(\delta_{ij} + \chi_{ij})E_j = \varepsilon_{ij} E_j$$

$$D_i = \varepsilon_{ij} E_j = -\varepsilon_{ij}\varphi_{,j}$$

Governing equation

$$D_{i,i} = \rho_f$$

$$\varepsilon_{ij}\varphi_{,ji} = -\rho_f$$

- Boundary conditions

$$(\mathbf{E}_2 - \mathbf{E}_1)\cdot \mathbf{n} = \frac{\sigma_f + \sigma_p}{\varepsilon_0}$$

$$\varepsilon_0(\mathbf{E}_2 - \mathbf{E}_1)\cdot \mathbf{n} = \sigma_f + (\mathbf{P}_1 - \mathbf{P}_2)\cdot \mathbf{n}$$

$$(\mathbf{D}_2 - \mathbf{D}_1)\cdot \mathbf{n} = \sigma_f$$

$$\varepsilon_1 \frac{\partial \varphi_1}{\partial n} - \varepsilon_2 \frac{\partial \varphi_2}{\partial n} = \sigma_f \quad \text{and} \quad \varphi_1 = \varphi_2$$

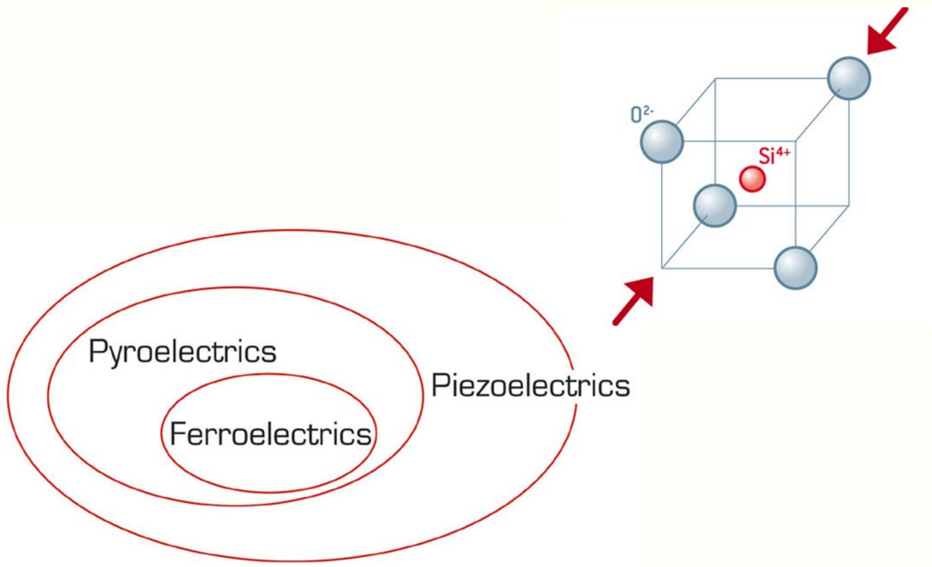
Boundary condition

$$(D_i n_i)_2 - (D_i n_i)_1 = \sigma_f$$

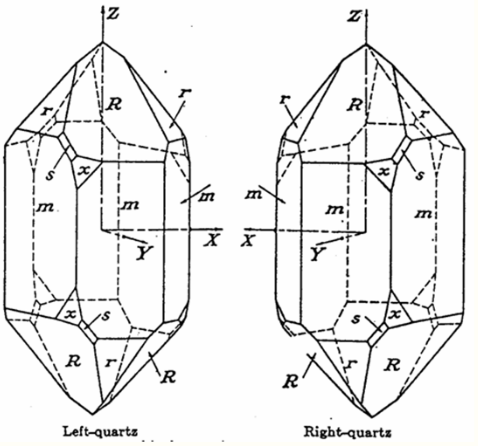
$$(\varepsilon_{ij}\varphi_{,j} n_i)_1 - (\varepsilon_{ij}\varphi_{,j} n_i)_2 = \sigma_f$$

and $\varphi_1 = \varphi_2$

Piezoelectricity



• Quartz



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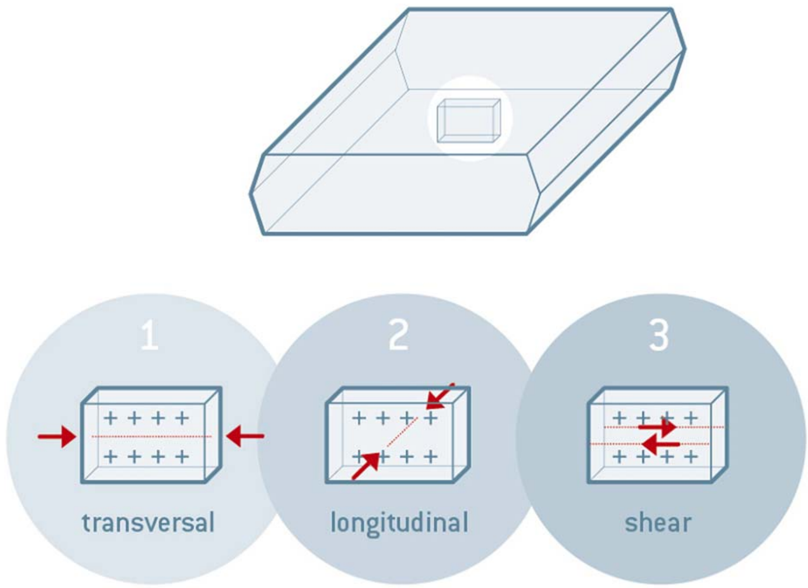

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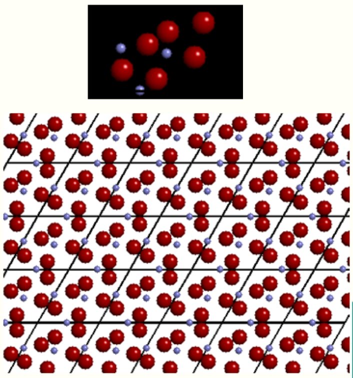
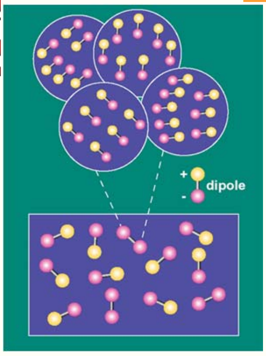
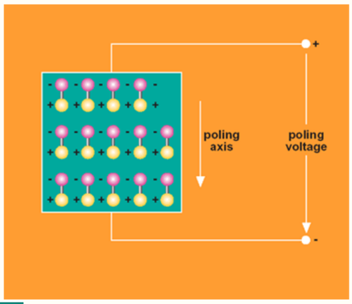


Quartz in rest state

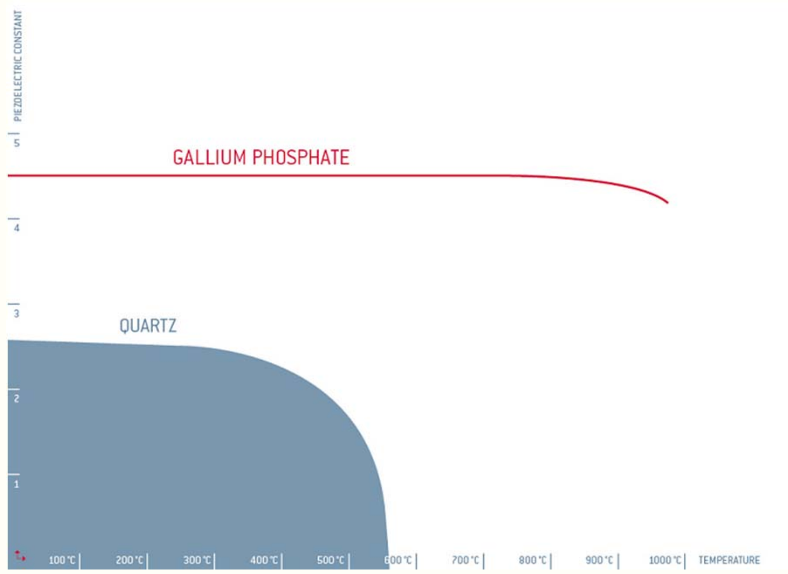


Force is applied



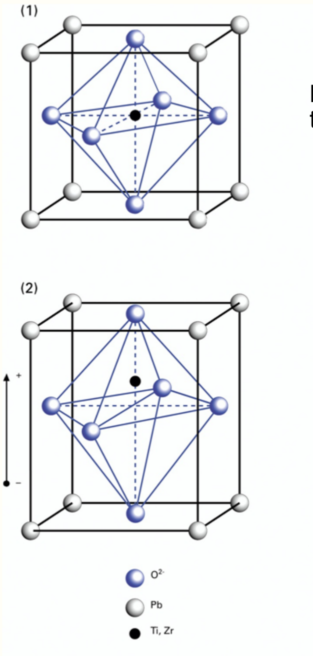
Potential is created

Temperature effect



•Ferroelectric material: PZT

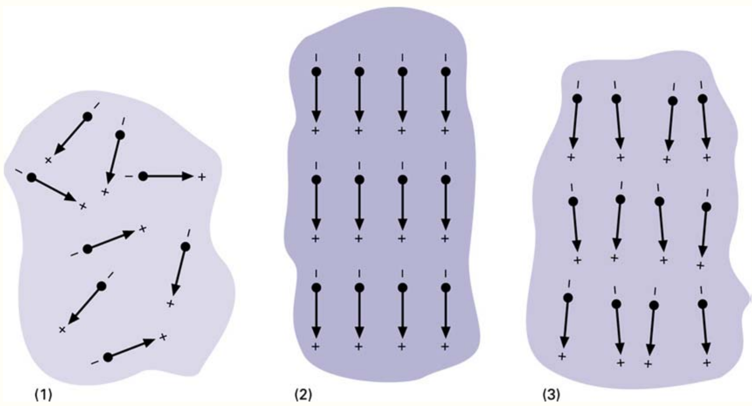
$PbTiO_3$, $PbZrO_3$



PZT unit cell above the Curie temperature

PZT unit cell below the Curie temperature

Poling of piezoelectric ceramic



Electric dipoles in domains: (1) unpoled ferroelectric ceramic, (2) during and (3) after poling

• Constitutive equations

$$T_{ij} = c^E_{ijkl} S_{kl} - e_{kij} E_k \qquad T_{ij}: \text{stress}$$

$$D_i = e_{ikl} S_{kl} + \varepsilon^S_{ij} E_j \qquad S_{kl}: \text{strain}$$

• Alternate forms of constitutive equations

$$S_{ij} = s^E_{ijkl} T_{kl} + d_{kij} E_k \qquad S_{ij} = s^D_{ijkl} T_{kl} + g_{kij} D_k \qquad T_{ij} = c^D_{ijkl} S_{kl} - h_{kij} D_k$$

$$D_i = d_{ikl} T_{kl} + \varepsilon^T_{ik} E_k \qquad E_i = -g_{ikl} T_{kl} + \beta^T_{ik} D_k \qquad E_i = -h_{ikl} S_{kl} + \beta^S_{ik} D_k$$

• Governing equations

$$Tij,i = \rho \ddot{u}_j \qquad c^E_{ijkl} u_{k,li} + e_{kij} \varphi_{,ki} = \rho \ddot{u}_j$$

$$D_{i,i} = 0 \qquad \Rightarrow \qquad e_{kij} u_{i,jk} - \varepsilon^S_{ij} \varphi_{,ij} = 0$$

• Boundary conditions

For a surface of discontinuity

$$n_i T^I_{ij} = n_i T^{II}_{ij} \qquad n_i T_{ij} = 0 \text{ for a traction free surface}$$

$$u^I_j = u^{II}_j \qquad u_j = 0 \text{ for a fixed surface}$$

$$n_i D^I_i = n_i D^{II}_i \qquad n_i D_i = 0 \text{ at an air-dielectric interface}$$

$$\varphi^I = \varphi^{II} \qquad \varphi = 0 \text{ short-circuited electrodes}$$

Material constants for PZT

$$\begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{bmatrix} = \begin{bmatrix} s^E_{11} & s^E_{12} & s^E_{13} & 0 & 0 & 0 \\ s^E_{12} & s^E_{11} & s^E_{13} & 0 & 0 & 0 \\ s^E_{13} & s^E_{13} & s^E_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & s^E_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & s^E_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & s^E_{66} = 2\left(s^E_{11} - s^E_{12}\right) \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} + \begin{bmatrix} 0 & 0 & d_{31} \\ 0 & 0 & d_{31} \\ 0 & 0 & d_{33} \\ 0 & d_{15} & 0 \\ d_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

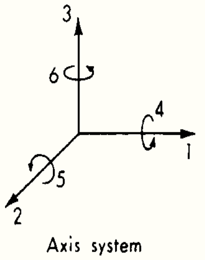
$$\begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & d_{15} & 0 \\ 0 & 0 & 0 & d_{15} & 0 & 0 \\ d_{31} & d_{31} & d_{33} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \end{bmatrix} + \begin{bmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{11} & 0 \\ 0 & 0 & \epsilon_{33} \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix}$$

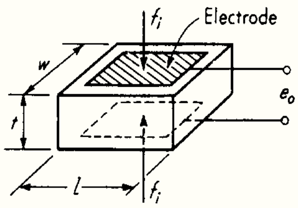
Signal Conditioning

• Piezoelectric transducers

$$g_{33} \equiv \frac{\text{field produced in direction 3}}{\text{stress applied in direction 3}} = \frac{e_0 / t}{f_i / (wl)}$$

$$d_{33} \equiv \frac{\text{charge generated in direction 3}}{\text{force applied in direction 3}} = \frac{Q}{f_i}$$





Charge generated by the crystal

$q = K_q x_i$, x_i :deflection

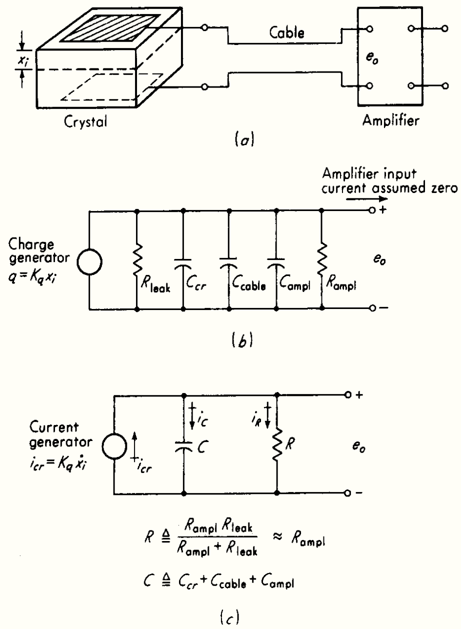
Current generated by the crystal

$$i_{cr} = \frac{dq}{dt} = K_q \frac{dx_i}{dt}$$

$$i_{cr} = i_C + i_R$$

$$e_o = e_C = \frac{\int i_C dt}{C} = \frac{\int (i_{cr} - i_R) dt}{C}$$

$$C\left(\frac{de_o}{dt}\right) = i_{cr} - i_R = K_q\left(\frac{dx_i}{dt}\right) - \frac{e_o}{R}$$

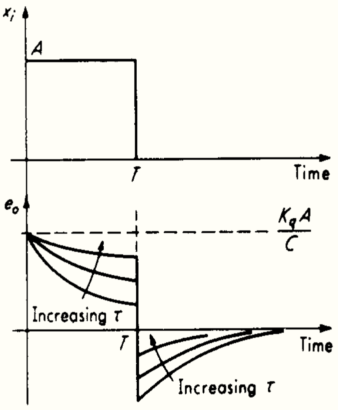


Frequency response function

$$x_i = X_i \exp(i\omega t)$$
$$e_o = V_o \exp(i\omega t)$$

$$\frac{V_0}{X_i} = \frac{(K_q / C) i\omega}{i\omega + (1/CR)} = \frac{i\tau\omega(K_q / C)}{i\tau\omega + 1}$$

$$\tau = CR$$



Step response

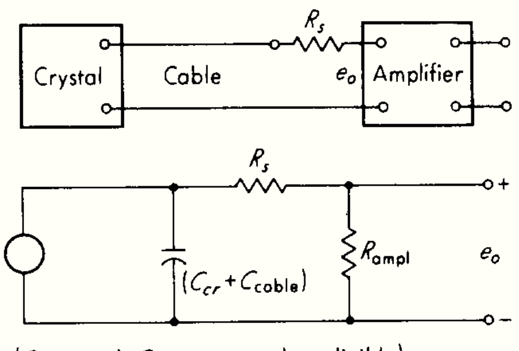
$$x_i = A \quad \text{for} \quad 0 < t < T$$
$$x_i = 0 \quad \text{for} \quad \mathrm{T} < \mathrm{t}$$

$$e_o = \frac{K_q A}{C} \exp(-t / \tau) \qquad 0 < \mathrm{t} < \mathrm{T}$$

$$e_o = \frac{K_q A}{C} [\exp(-T / \tau) - 1] \exp[-(t - T) / \tau] \qquad \mathrm{T} < \mathrm{t}$$

Use series resistor

- sacrifices sensitivity



(R_{leak} and C_{ampl} assumed negligible)

$$K \triangleq \frac{K_q}{C} \left(\frac{R_{ampl}}{R_{ampl} + R_s} \right)$$

$$\tau \triangleq (R_{ampl} + R_s) C$$

$$C \triangleq C_{cr} + C_{cable}$$

- Charge Amplifiers

Output voltage change

$$\Delta v_o = \frac{-V_c \Delta C}{C_f}$$

or

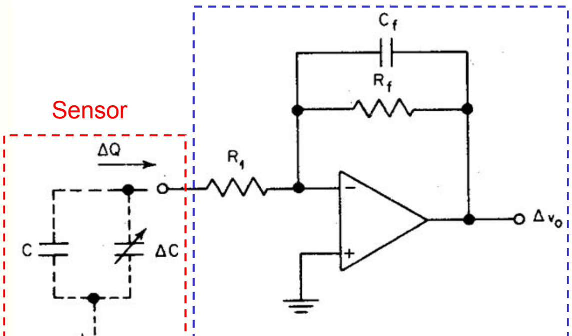
$$\Delta v_o = \frac{-\Delta Q}{C_f}$$

Lower cutoff frequency (-3dB)

$$f_{cp1} = \frac{1}{2\pi R_f C_f}$$

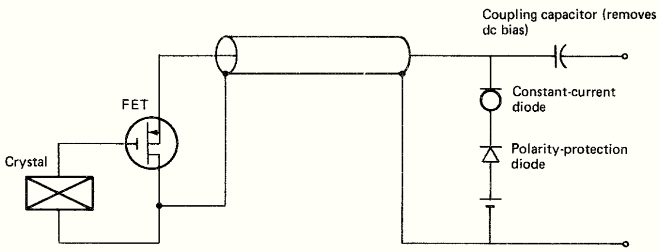
Upper cutoff frequency (-3dB)

$$f_{cp2} = \frac{1}{2\pi R_1 C}$$



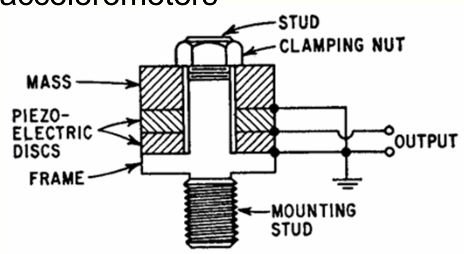
$$\frac{e_o}{e_i} = \frac{-R_f}{R_1(1+i\omega C_f R_f)}$$

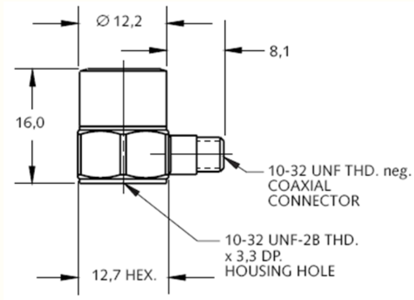
- Impedance converter



Applications

- Piezoelectric accelerometers





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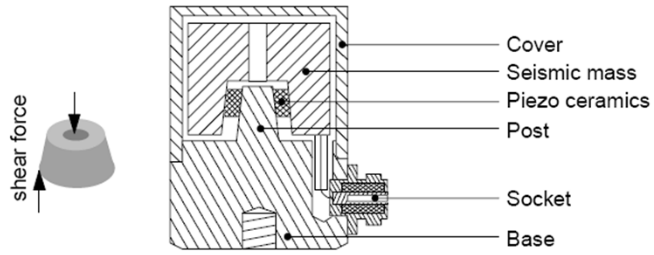
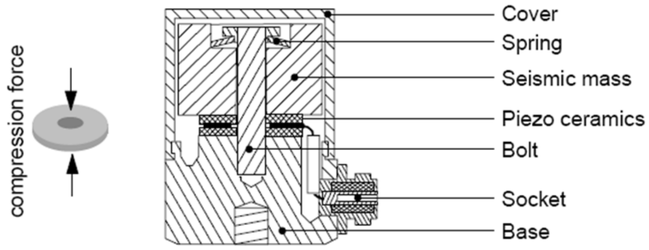
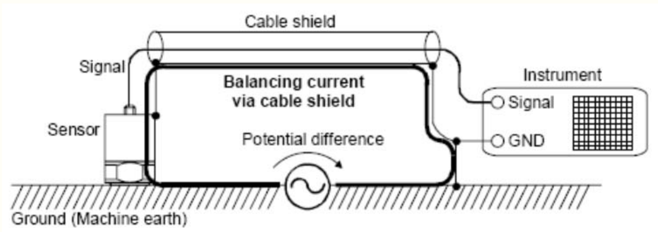
Shear Design:

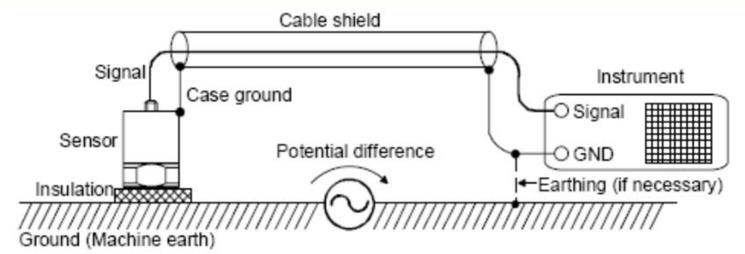


Compression Design:



Model (Single axis or triaxial)		Single axis linear
Range	g	±2000
Sensitivity	pC/g	-10.000
Frequency Range	Hz	5...10000
Resolution, Threshold	mgrms	1
Transverse Sensitivity	%	1.5
Non linearity	% FSO	±1
Shock	g	5000
Temp. coef. of sensitivity	%/˚C	0.13
Operating temperature range	˚C	-70...250
Housing/Base		stainless steel
Sealing		hermetic (IP68)
Ground isolation		No
Mass	g	14.5
Connector		10-32 neg.
Diameter	mm	16
Height	mm	12.19
Mounting		stud/wax
Mounting thread		10-32 UNF x 3,3



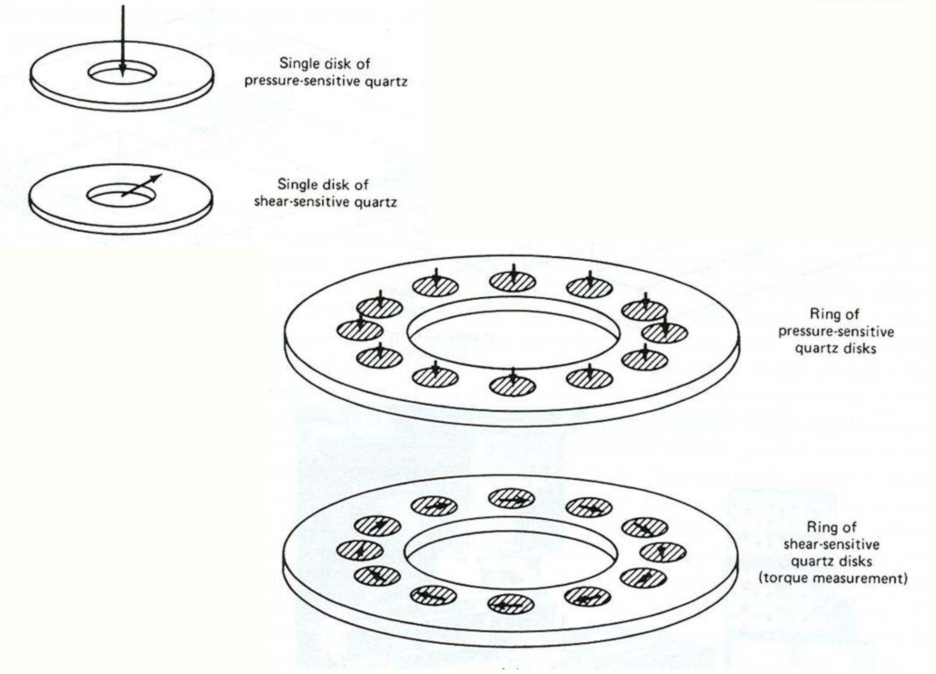


• Quartz torque sensor

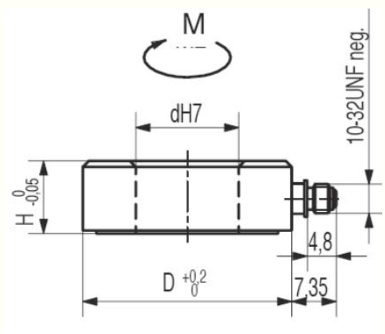
The torque sensor consists of two steel disks, between which a ring is fitted which contains several shear-sensitive quartz plates. The crystal axes of the quartz plates are oriented tangentially to the peripheral direction and therefore yield a charge exactly proportional to the applied torque.

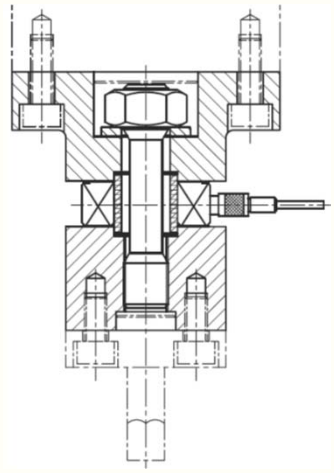
Application examples

- Adjusting torques of pneumatic screw-drivers
- Testing of screw connections
- Calibration measurements of manual torque wrenches
- Testing torsion of springs
- Measurements of friction clutches
- Measuring starting torques, variations in synchronization and torsional vibrations of fractional horsepower and stepping motors.
- Testing of rotary switch



The torque sensor must be mounted under elastic preload as the torque must be transmitted by *static friction* onto the front parts of the sensor.





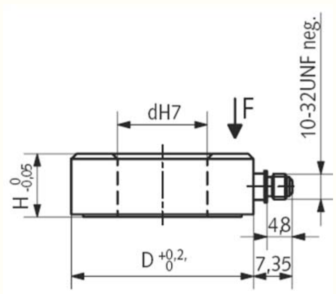
Testing of rotary switch

• Quartz Load Washers

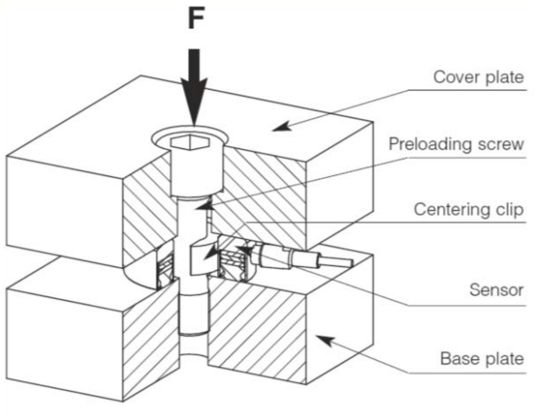
The force to be measured acts through the cover and base of the tightly welded steel housing on the quartz sensing elements. Quartz yields an electric charge proportional to the mechanical load.

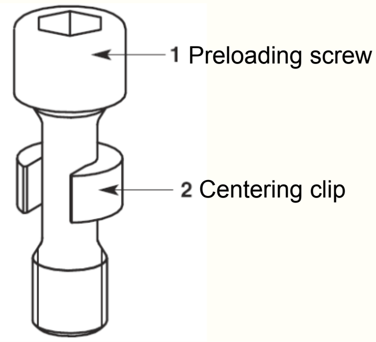
Application examples

- forces in spot welding
- forces in presses
- force variations in bolted connections under high static preload
- shock and fatigue resistance
- cutting and forming forces
- forces in railroad brakes
- impact forces



Mounting
The load washers must be installed between two plane-parallel, rigid and fine-machined (preferably ground) faces. This is necessary to achieve a good load distribution on one hand and a wide frequency response on the other hand. The load washers should always be installed under preload.





• 3-Component Quartz Crash Force Elements

Specification

Measuring Range	Fx	kN	0...500
	Fy, Fz	kN	±100
Non linearity		% FSO	<±0.5
Natural Frequency	fnx	kHz	≈4
	fny, fnz	kHz	≈1.7
Operating temperature range		˚C	0...50
Sealing			IP65

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Application

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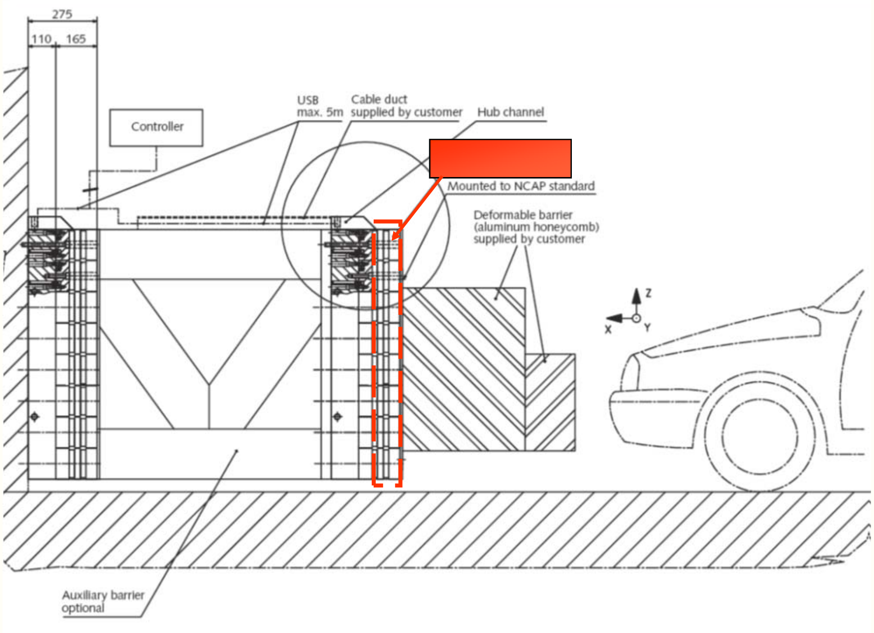

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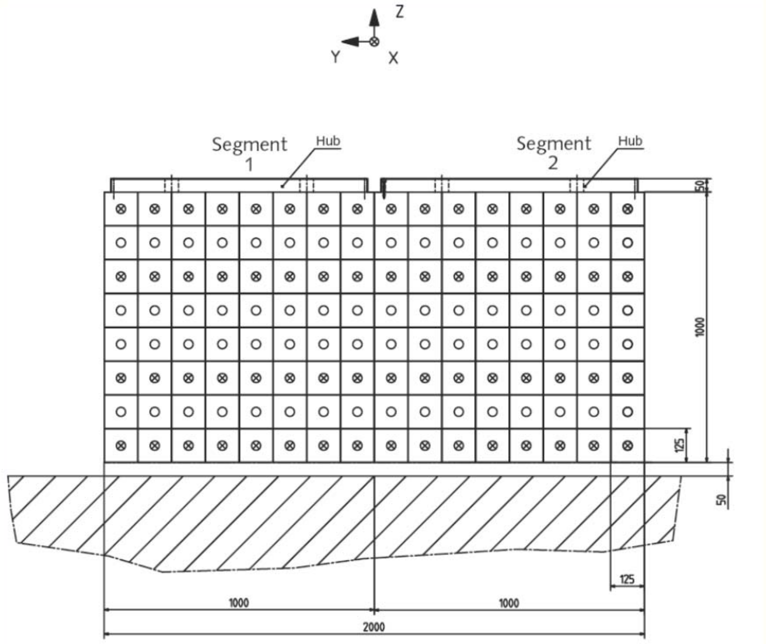


- Piezoelectric pressure sensors

Specification

Type 6005		
Measuring range	bar	0...1000
Overload	bar	1500.0
Sensitivity	pC/bar	10
Natural Frequency	kHz	≈140
Non linearity	% FSO	<±0.8
Operating temperature range	˚C	-196...200
Acceleration sensitivity	bar/g	<0.001
Thread		Without thread
Cooling		not cooled
Diameter	mm	5.5
Length	mm	6
Connector		M4x0,35 neg.

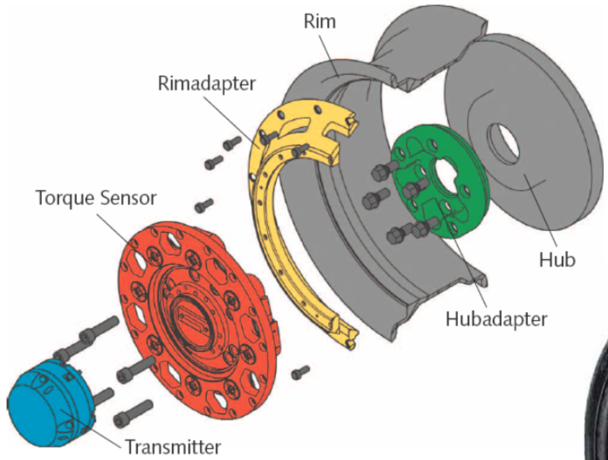
• Dynamometer

- Cutting force measurement

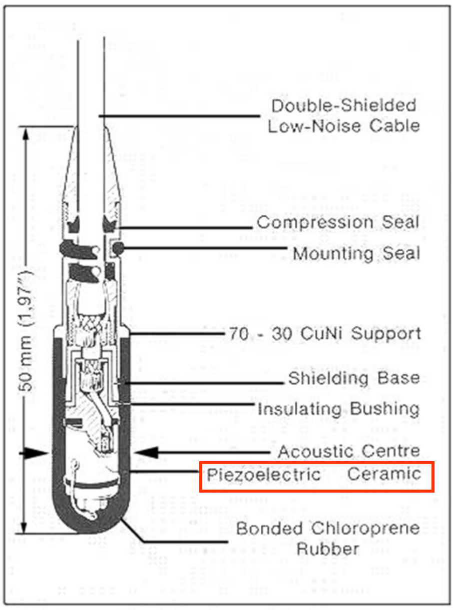
Calibration			calibrated
Measuring Range	Fx, Fy	kN	±20
	Fz	kN	±30
	Mz	N·m	±1100
Speed		1/min	max.5000
Sensitivity	Fx, Fy	mV/N	≈-0.5
	Fz	mV/N	≈-0.33
	Mz	mV/N·m	≈9
Natural Frequency		kHz	≈1
Operating temperature range		˚C	0...60
Diameter		mm	156
Height		mm	55
Mass		kg	7.5
Connection			Non-contacting
Sealing			welded/epoxy (IP67)

• Torque Wheel-Sensor

Model			Piezoelectric
Measuring Range	My	kN·m	±3
Natural Frequency	fny	kHz	1.1 Natural Frequency fny: free
Mass		kg	4.4
Maximum r.p.m.		1/min	2200 Max. speed ≈250 km/h
Crosstalk	Fy → My	N·m/kN	<±2
Offset/Variation	Fz → My	N·m/kN	<±2
Non linearity		% FSO	<±1
Diameter		mm	289
Hysteresis		% FSO	≤1
Sealing			IP65



- Hydrophone



Schematic drawing of hydrophone construction, Type 8103